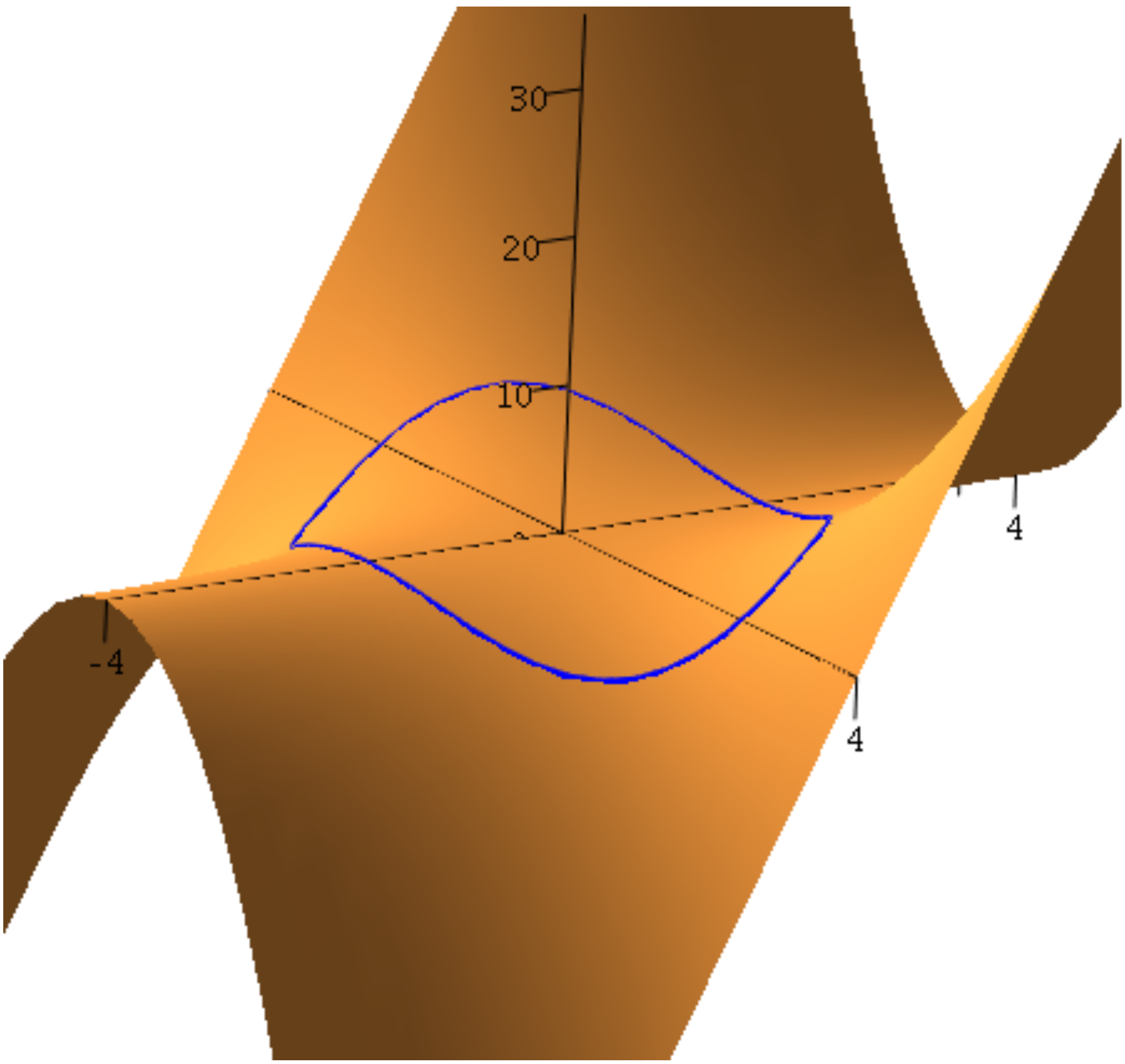
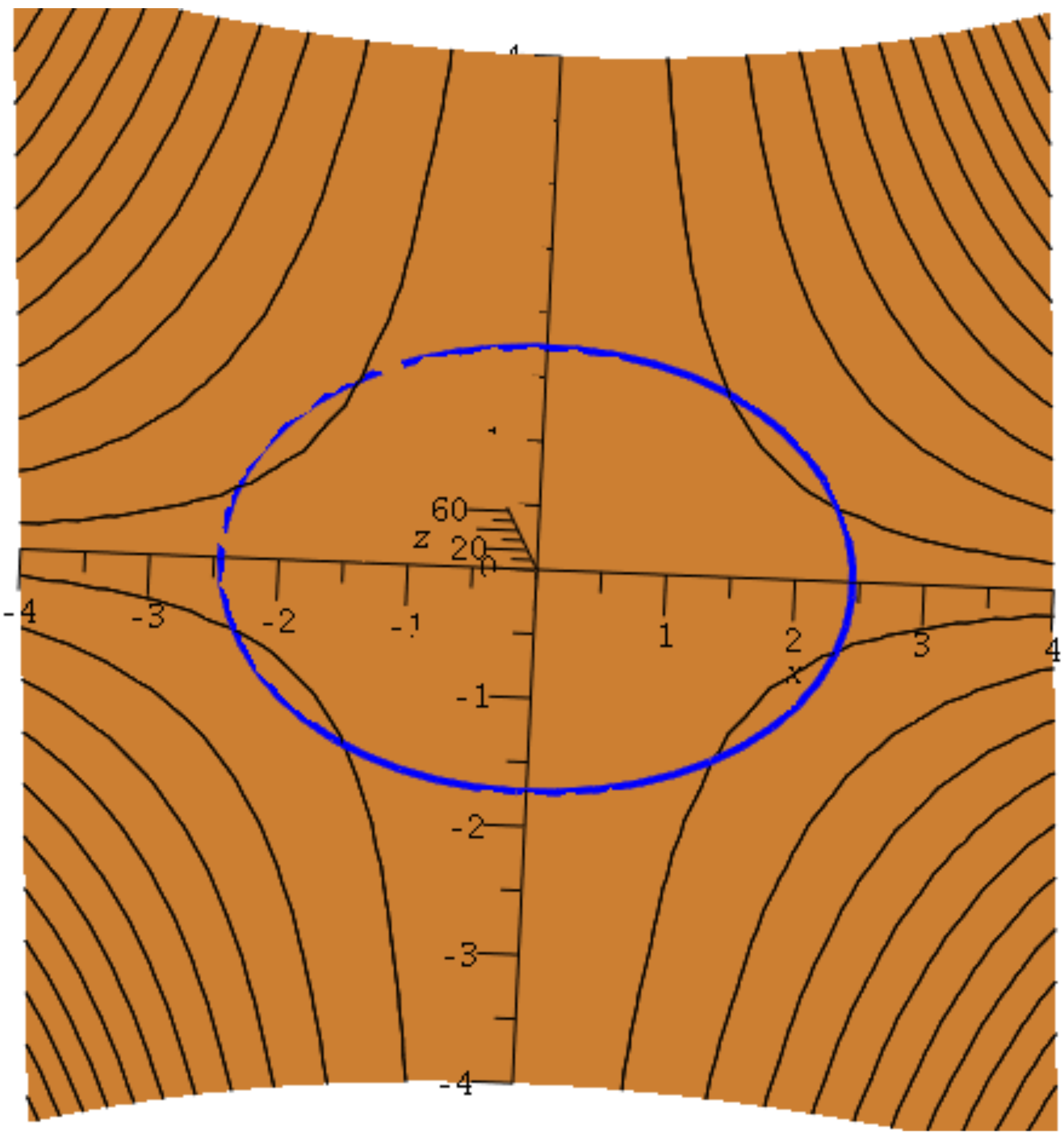
Section 15.8

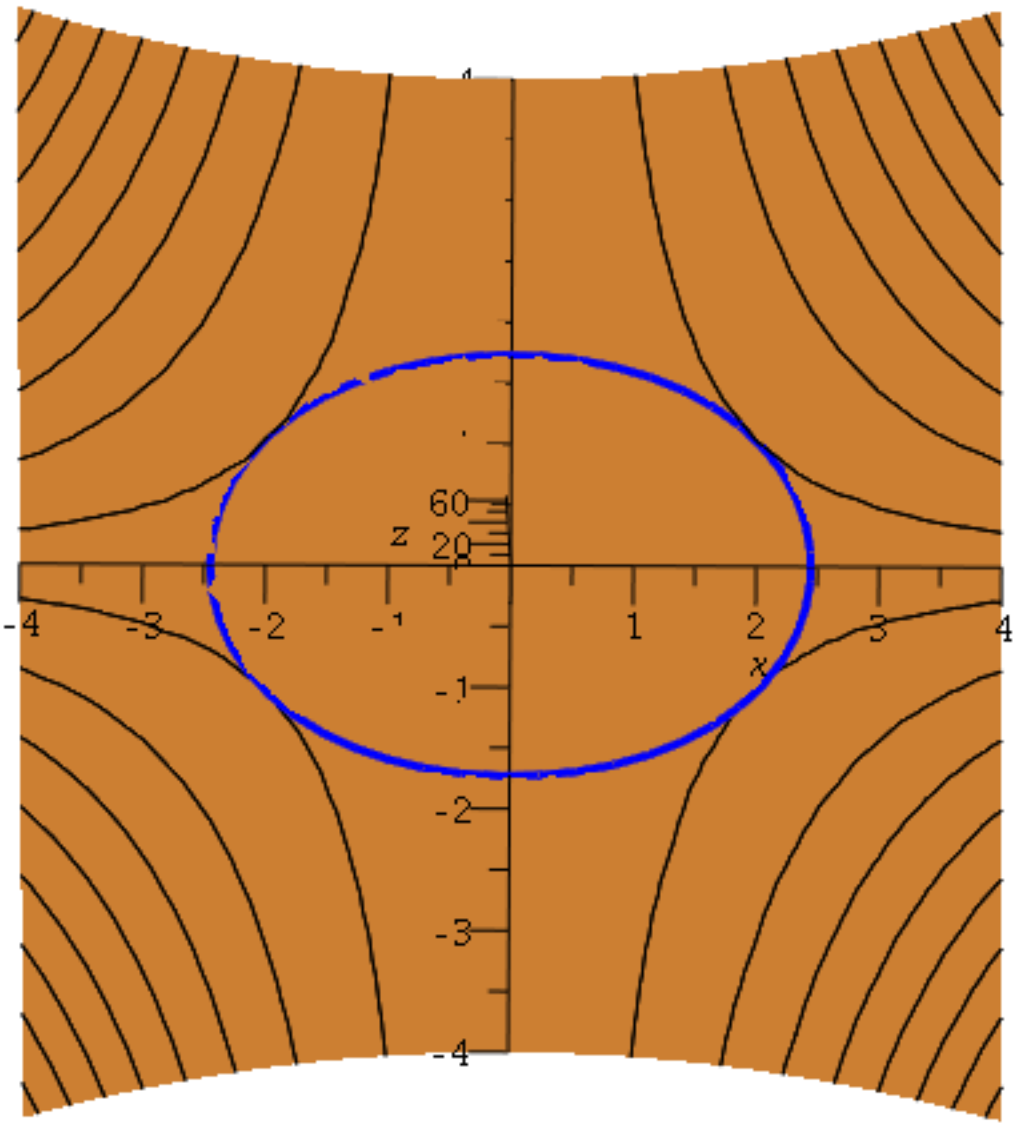
Suppose we want to consider the function $f(x, y) = x^2 \cdot y$ ***with the restriction that*** $x^2 + 2y^2 = 6$***. Graphically f(x,y) is the gold surface below. We are asking to find max and min values of the gold function, along the blue line (not inside the blue line but ON it).***



We can use the level curves of f(x,y) to analyze the situation. In the picture below notice that
(1) the level curves which do not interact with the blue line at all tell us nothing about where the max/min along blue occur and
(2) the level curves which intersect the blue line cannot be max or min since traveling away from that point along the blue line in one direction will produce larger f(x,y) values while traveling in the other direction along blue will produce smaller f(x,y) values.



However, when a level curve is tangent to the blue line, there could be a max or a min at that point. In these cases that one point is the only one on the level curve, and all other values of the blue line near that point are on one side or the other of the level curve. So the point would be a max or a min.



To have the level curve of f(x,y) be tangent to the blue line would mean that the gradients of both would be parallel. This is how we derive the equation to find the max/min: $\nabla f = \lambda \nabla g$.