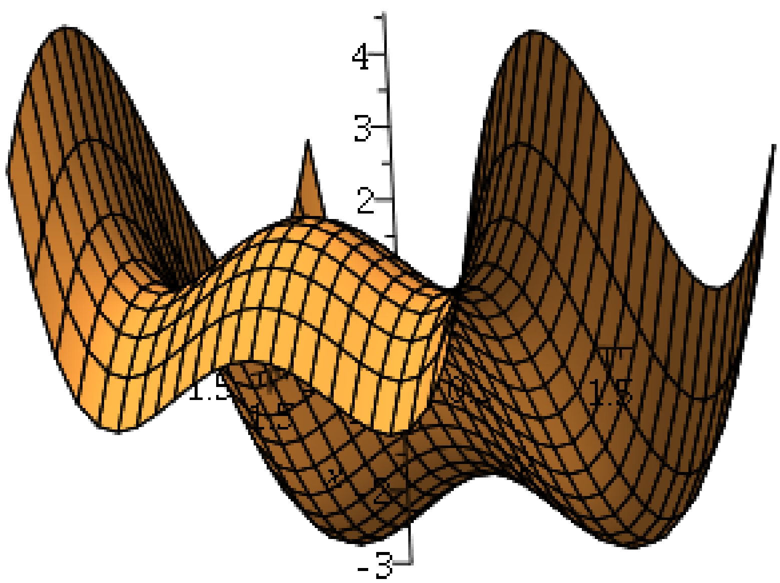
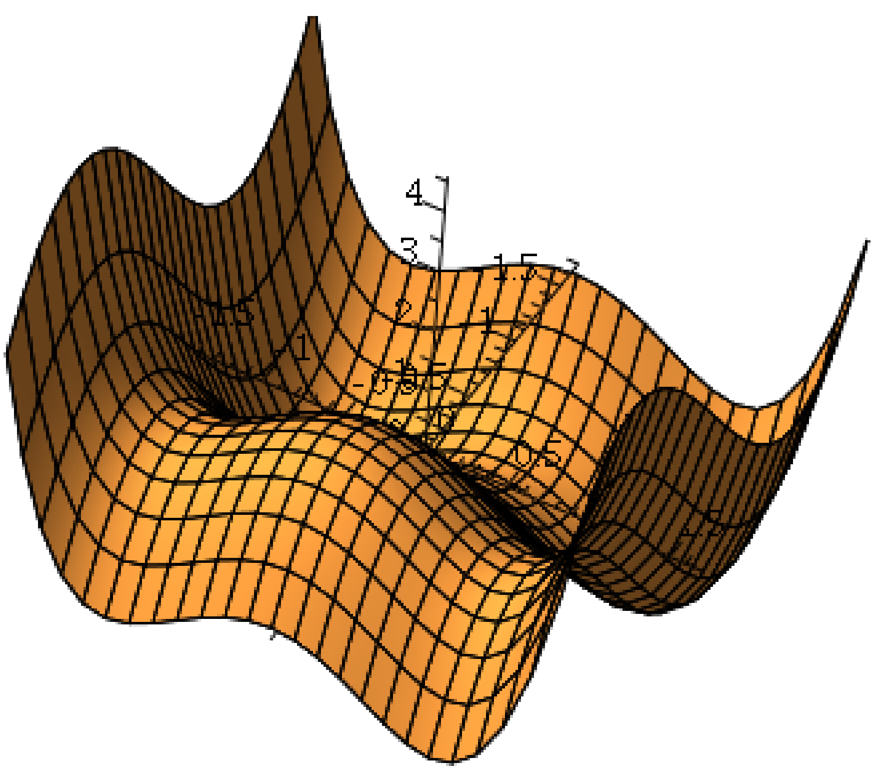
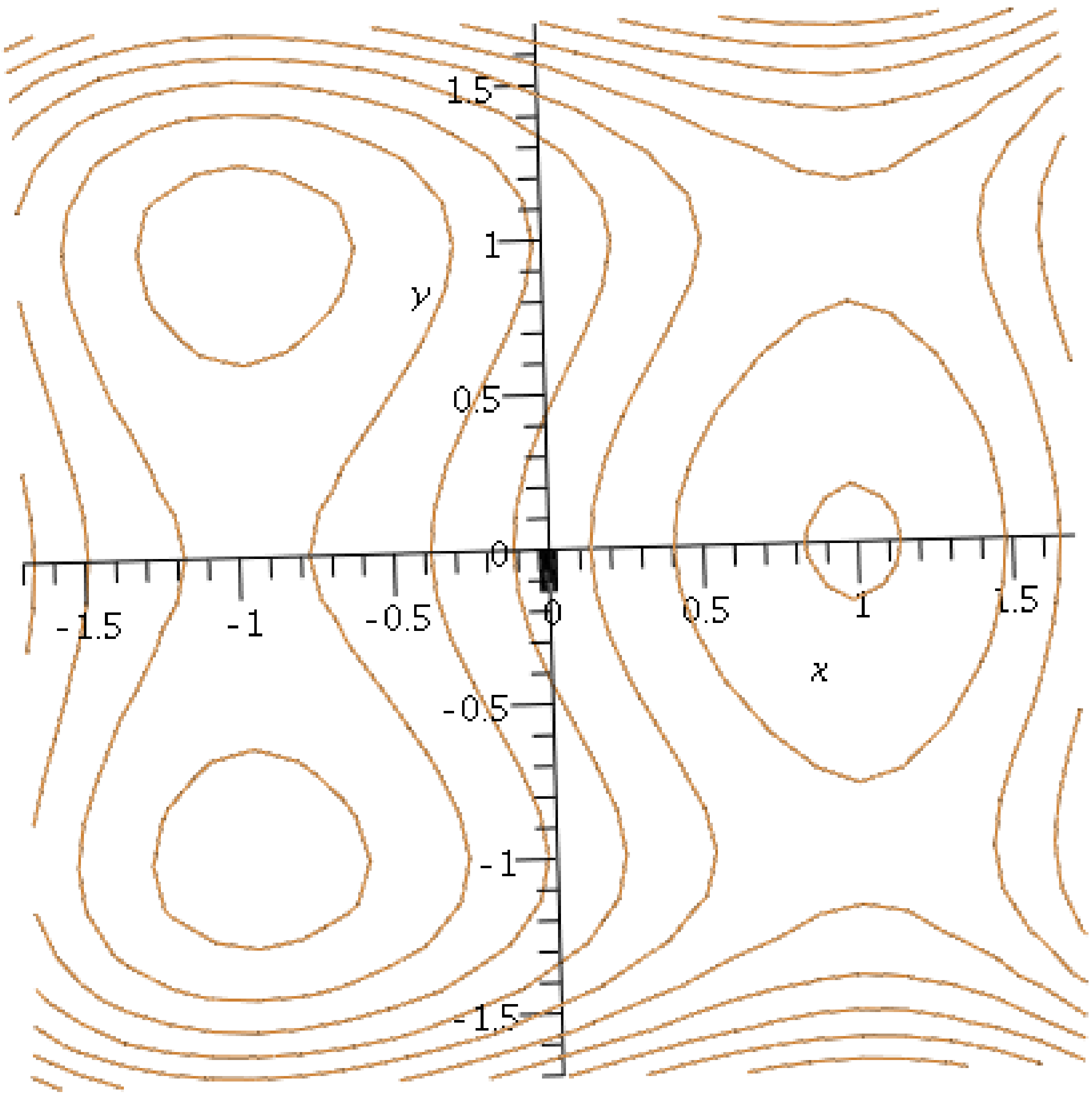
Section 15.7

Here are two views of the function $f(x, y) = 3 \cdot x - x^3 - 2 \cdot y^2 + y^4$ ***. There is one local max and two local mins, as well as several saddle points.***

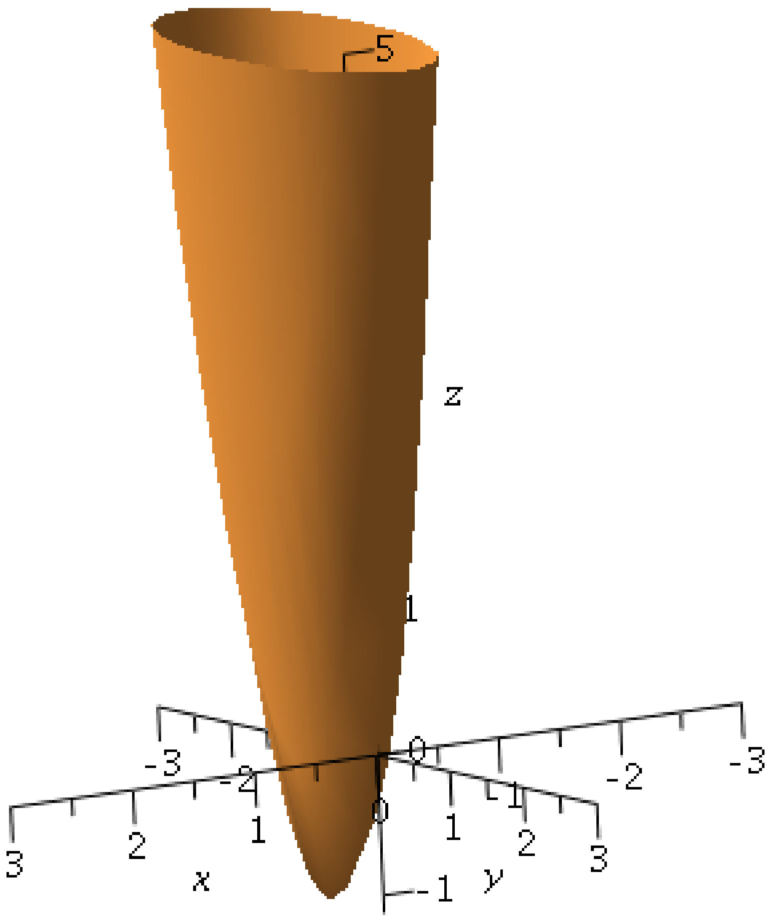




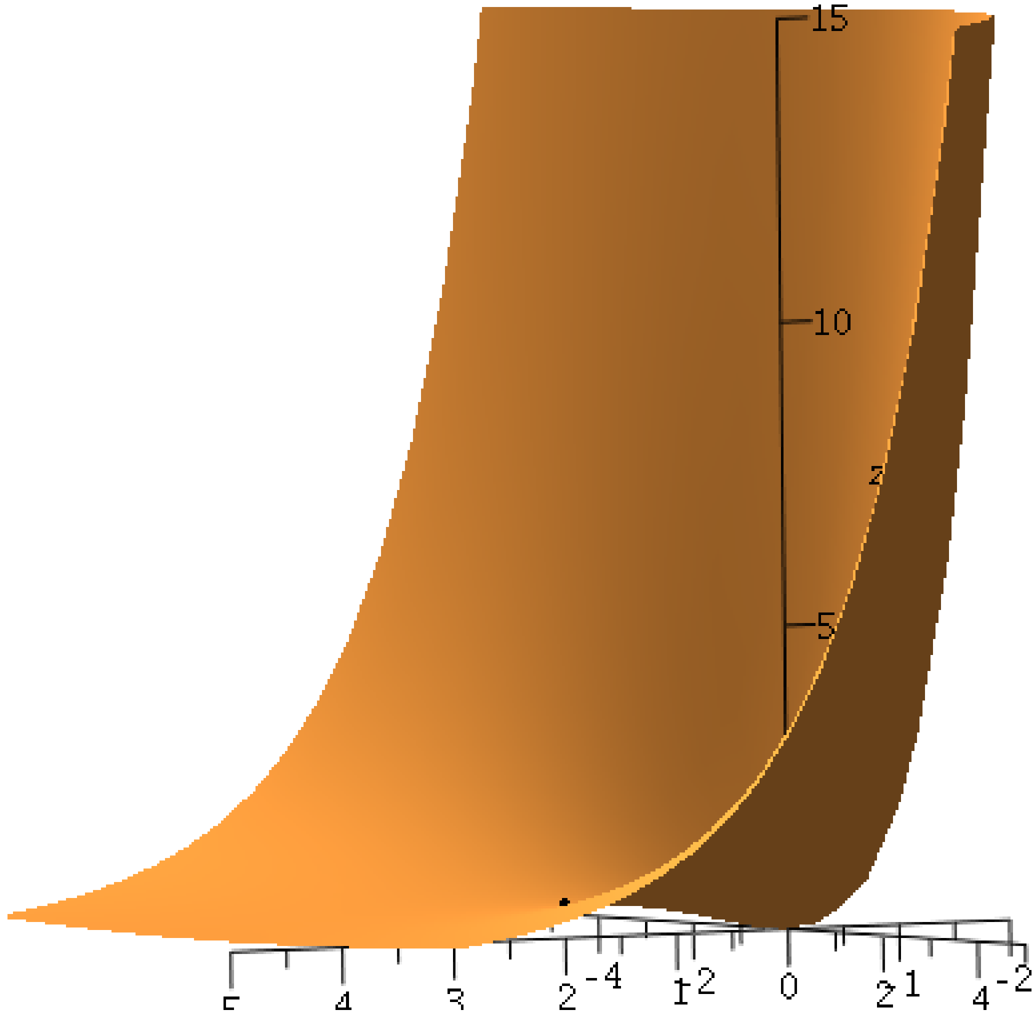
Here is the level curve picture for this function. Notice what the max and min look like in the level curves.



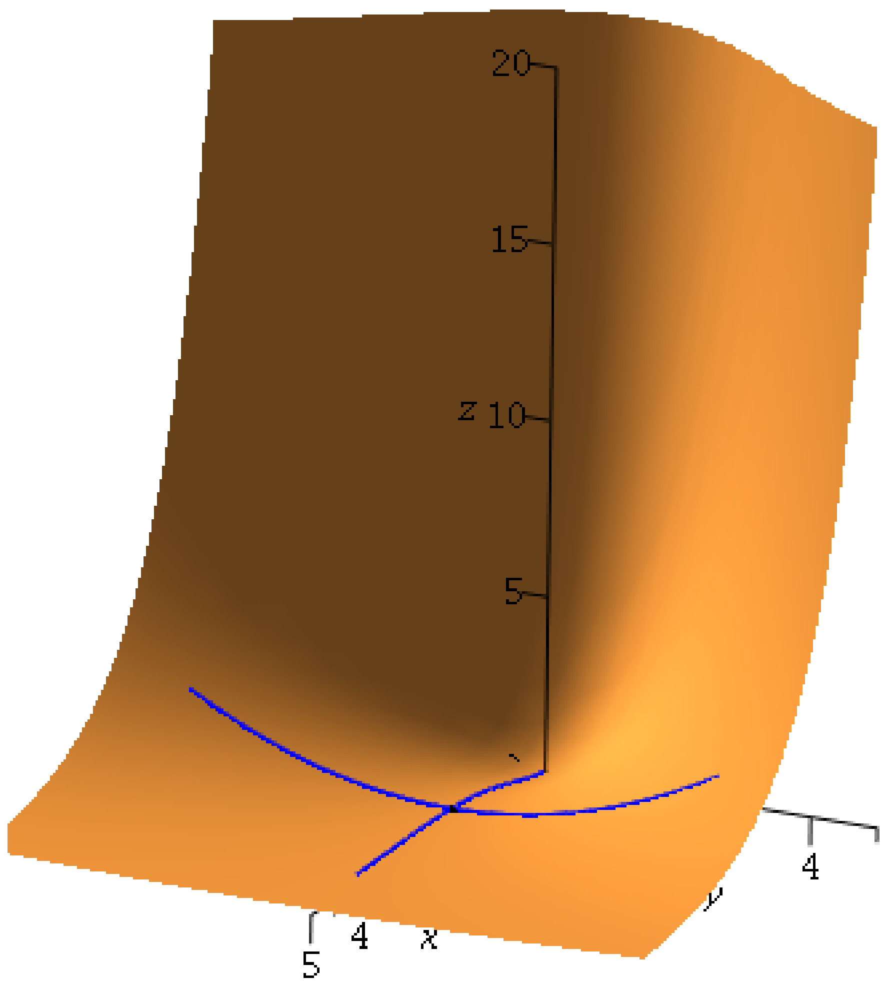
Here is a function $f(x, y) = 11 \cdot x^2 - 2 \cdot x \cdot y + 2 \cdot y^2 + 3\,y$ ***which has a minimum at (***$-\frac{1}{14}$, $-\frac{11}{14}$ ***).***



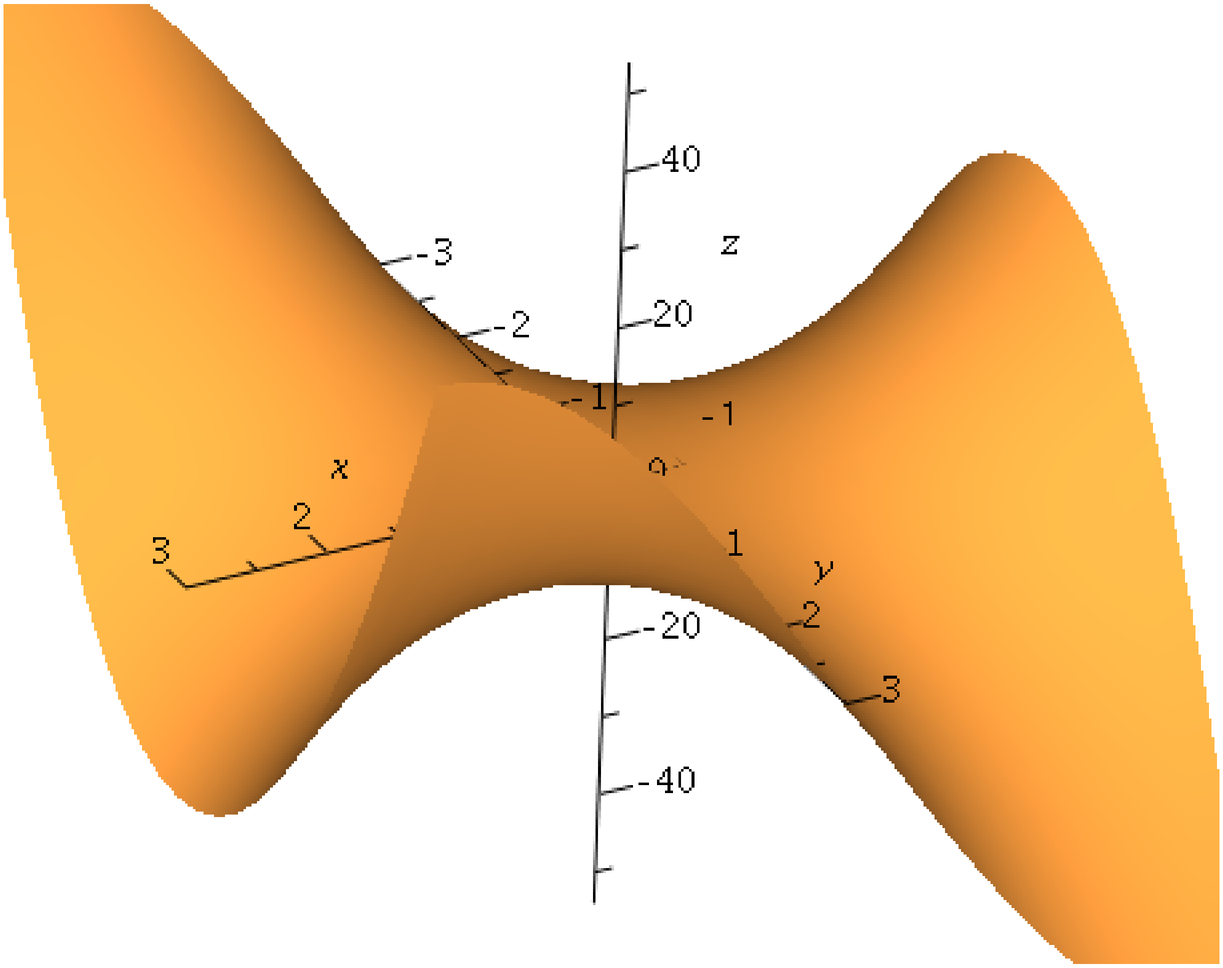
The function $f(x, y) = (x^2 + y^2) \cdot e^{-x}$***. There are two critical points, one at (0,0) and one at (2,0).***



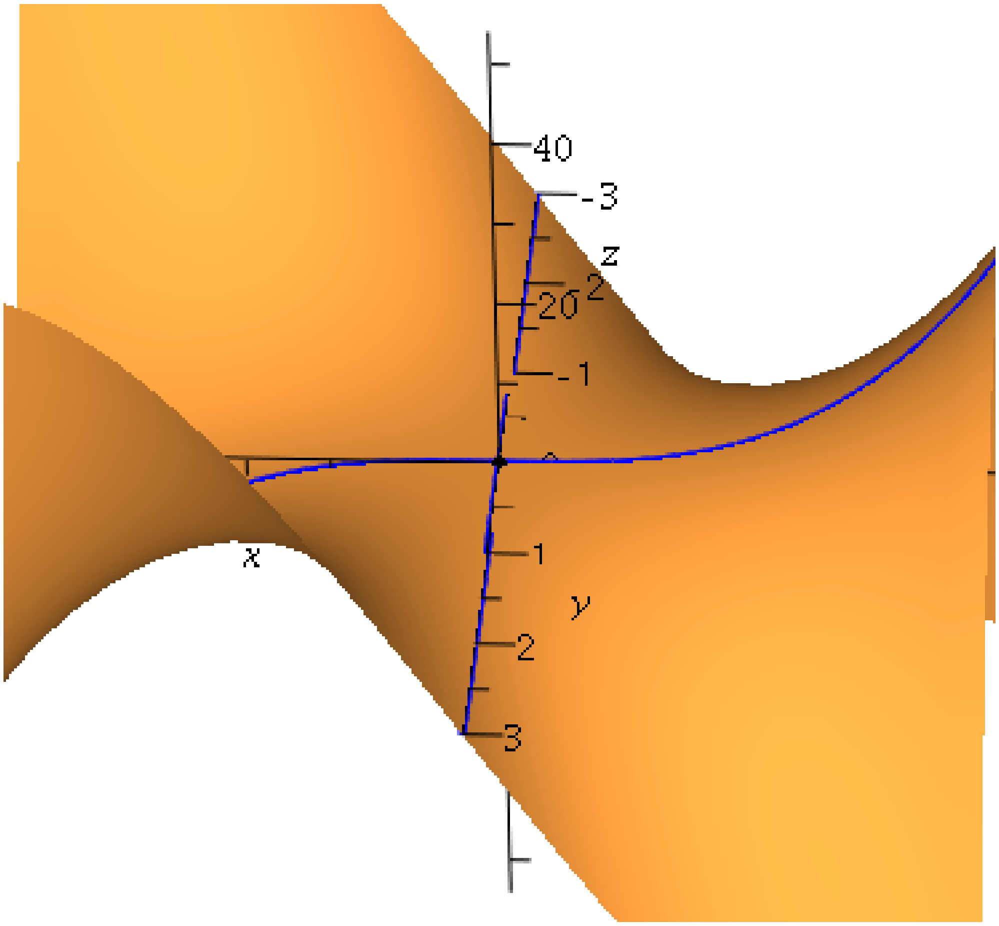
The point (0,0) is a minimum but the point (2,0) is a saddle point. The paths coming into (2,0) from the x and y direction are in blue below. You can see that if you fix x=2 and vary y, the point appears to be a minimum but if you fix y=0 and vary x the point appears to be a maximum.



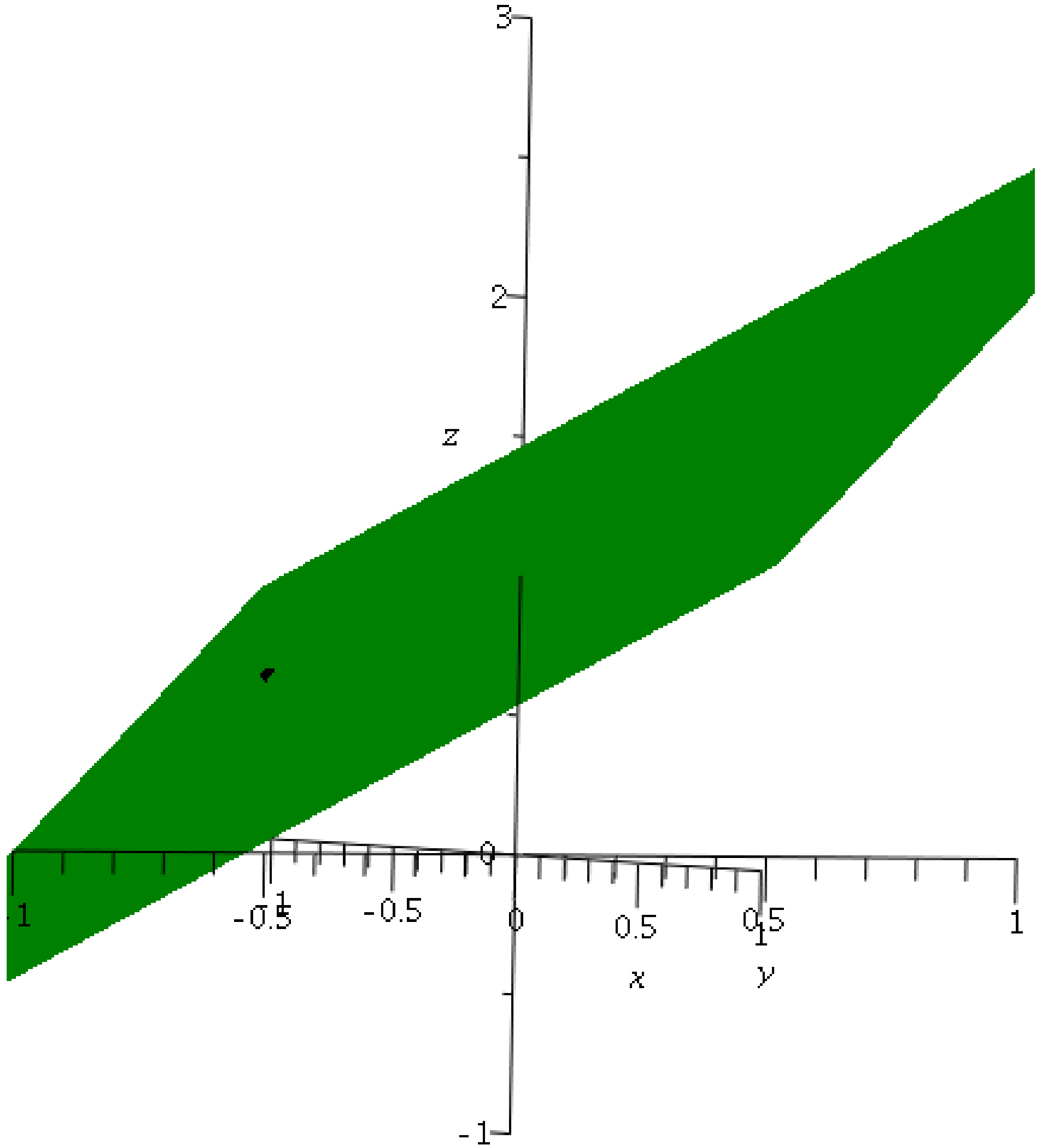
The function $f(x, y) = 3\,x \cdot y^2 - x^3$***. This function has a critical point at (0, 0).***



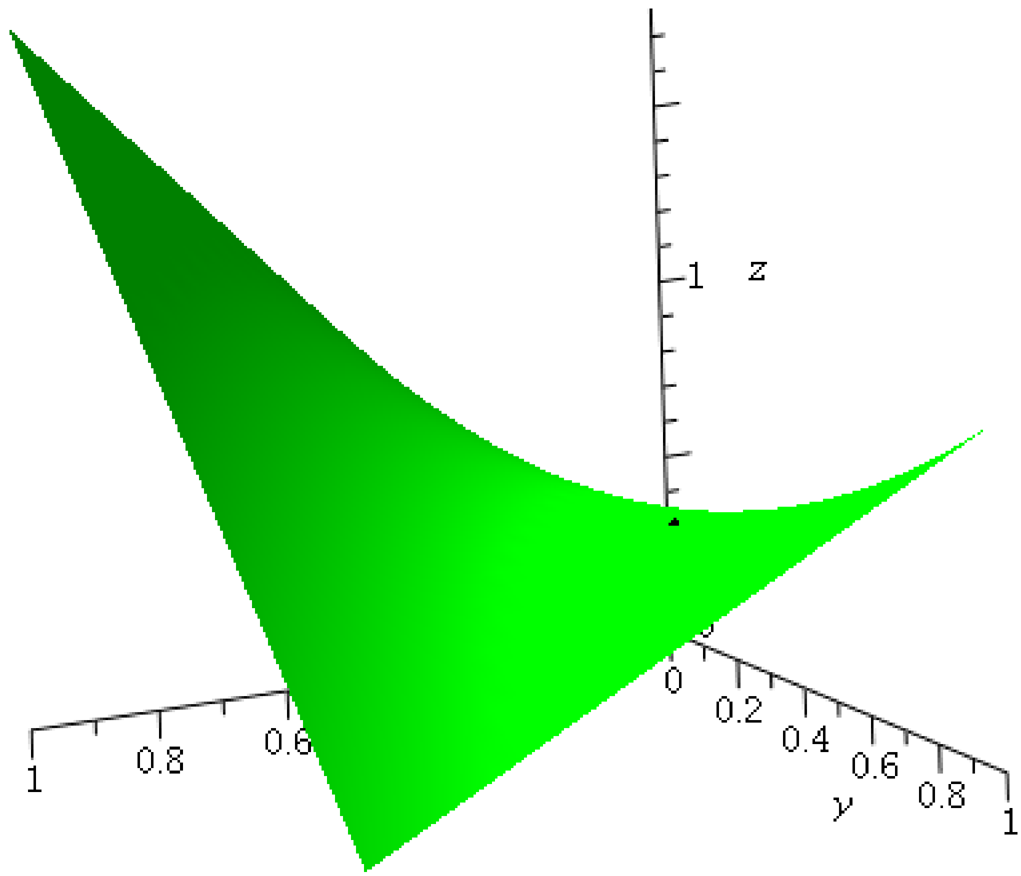
The paths as we come into the origin along the x and y axes confirm the 2nd Derivative test, that this point is a saddle point. Notice if we fix y=0 and vary x, the path traced is similar to the $y = x^3$ ***function.***



The function $f(x, y) = x + y + 1$. The point (1,0,0) in black is the point on the plane closest to (1,0,0). We used techniques of optimization problems to find this point.



The function $f(x, y) = 2x + y - 3xy$ ***with one critical point*** $\left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$***. The absolute max and min occur on the boundaries.***



z
1
1
0.8
0.6
0
0.2
0.4
0.6
0.8
1
y