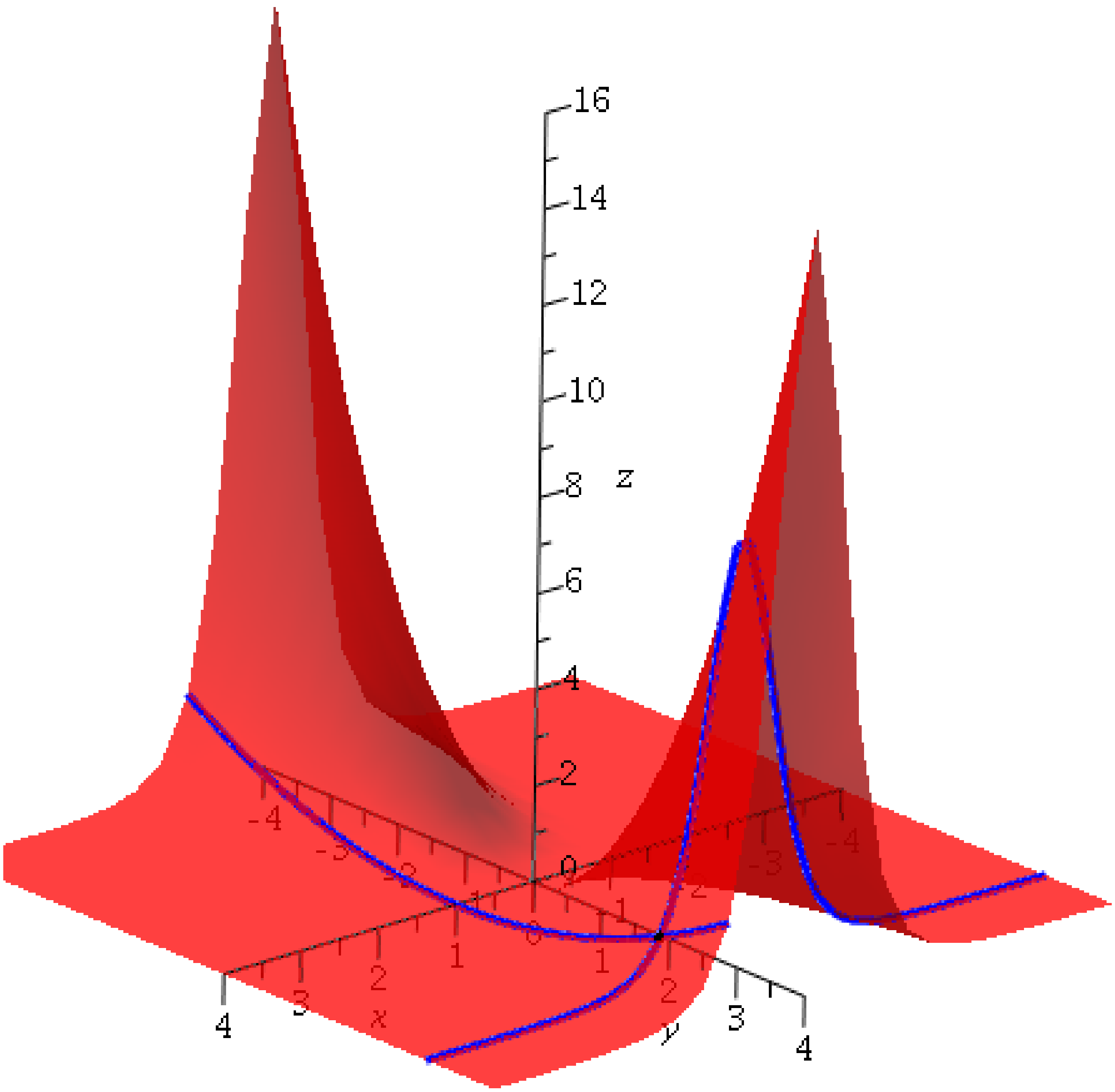
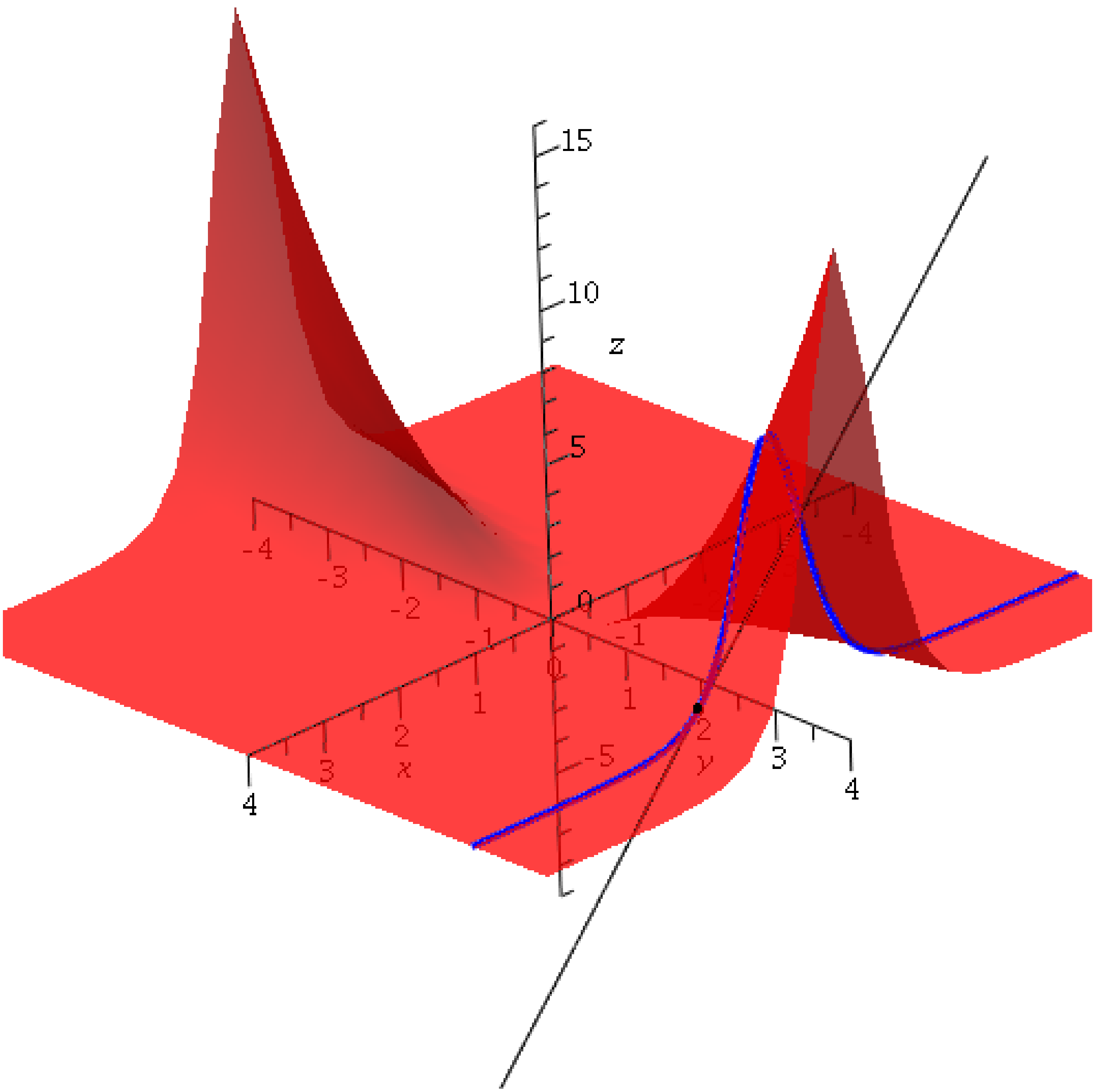
Section 15.3

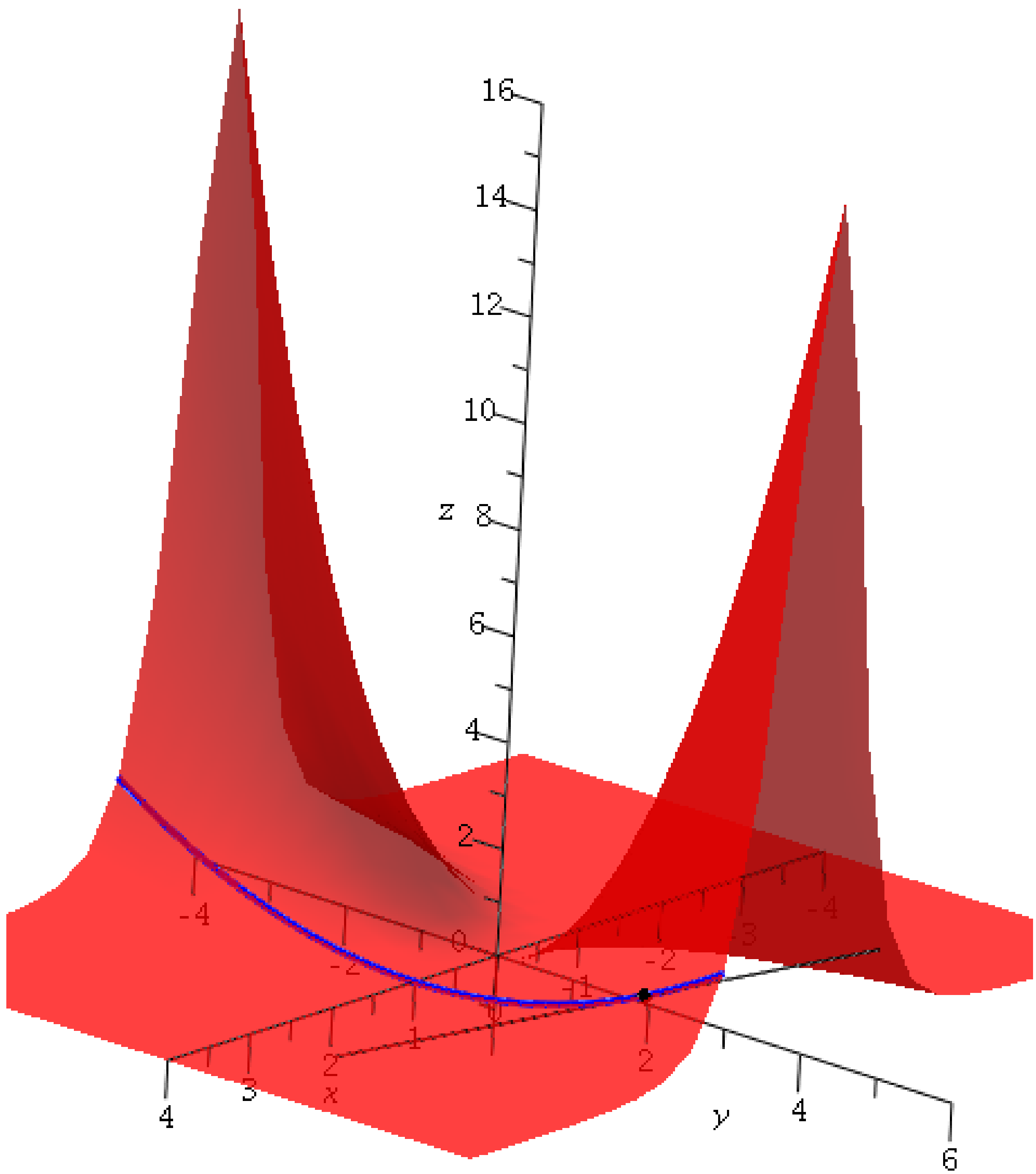
The function $f(x, y) = \dfrac{y^2}{(1+x^2)^3}$ ***with space curves in blue representing travel in the x and y direction toward the point (1,3).***



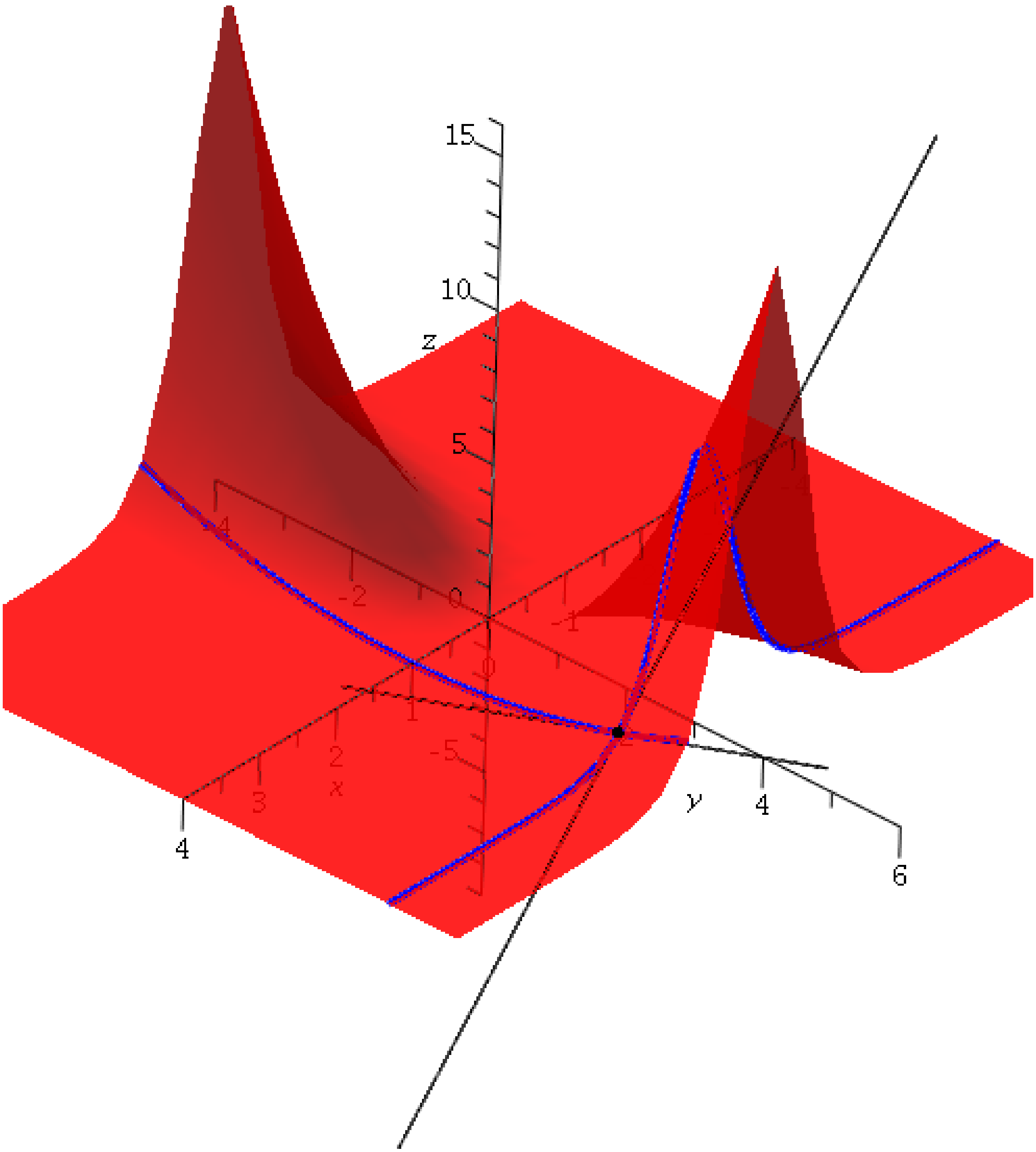
If we fix y and vary x, we follow the blue line below toward the point (1, 3). The black line represents the tangent line to the space curve. The slope of this tangent line will be $\frac{\partial f}{\partial x}(1, 3)$.



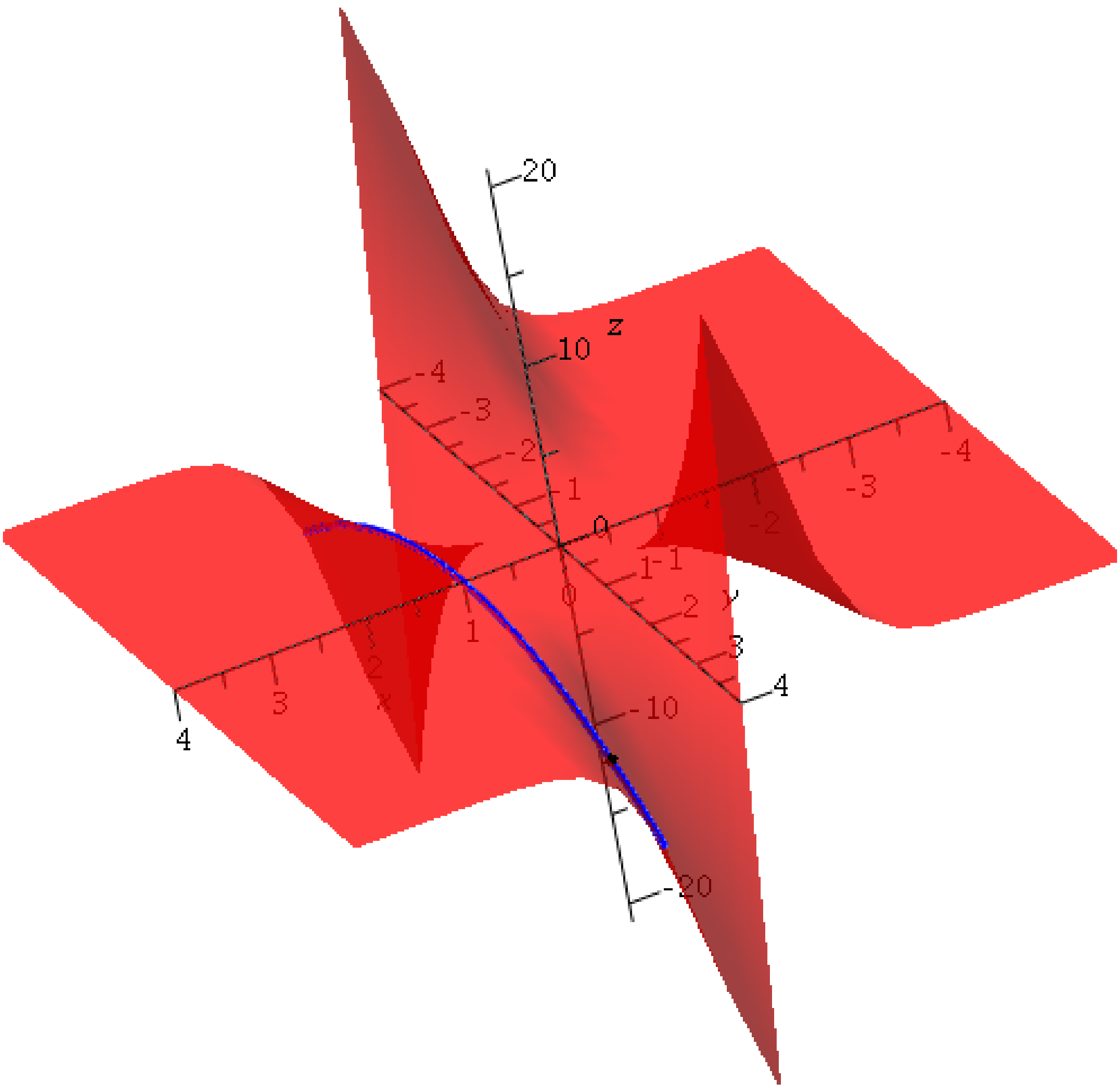
Similarly, if we fix x and vary y, we follow the blue line below toward the point (1,3). The black line again represents the tangent line to this space curve. The slope of this tangent line will be $\frac{\partial f}{\partial y}(1,3)$.



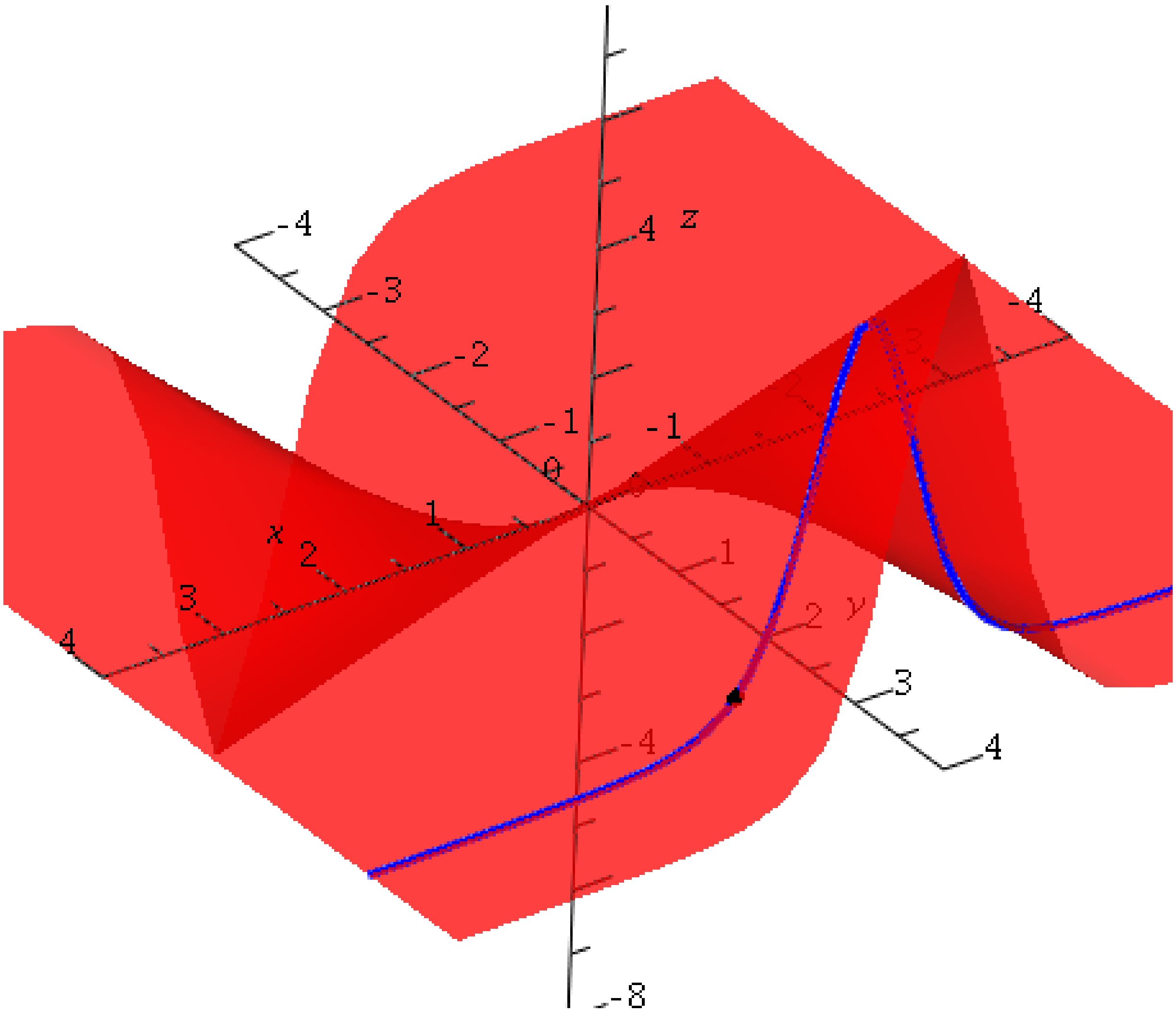
This picture shows the tangent line in both the x and y direction. The slope of these gives both partial derivatives.



The surface plotted below is the partial derivative of f(x,y) with respect to x. The partial is given by the expression $\frac{\partial f}{\partial x}(x, y) = \frac{-6 \cdot x \cdot y^2}{(1+x^2)^4}$***. The space curve represents how the partial with respect to x is changing in the y direction. (This change will be one of the mixed partial derivatives.)***



The surface plotted below is the partial derivative of f(x,y) with respect to y. The partial is given by the expression $\frac{\partial f}{\partial x}(x, y) = \frac{2 \cdot y}{(1 + x^2)^3}$***. The space curve represents how the partial with respect to y is changing in the x direction. (This will be the other mixed partial derivatives.)***



Notice that if you consider the slope of the space curves on each of the last two pictures at the point (1,3), which is the black dot, the slope looks the same. Clairaut's Theorem tells us that this must be true (the mixed partial derivatives are equal).