Math 215: Linear Algebra

Problem Set 11 : Due September 29

(28 points) Make sure you are familiar with the Academic Honesty policies for this class, as detailed on the syllabus. All work is due on the given day by 3 PM Grinnell Time, or 7 PM if you LaTeX the assignment. **For this assignment, show all of your computational work.**

1. (4 points) Suppose A is an *invertible* matrix and B and C are arbitrary matrices. Prove that if $AB = AC$, then $B = C$. (You might want to compare this to Problem Set 9, Problem 4.)

2. Take the system of linear equations:
$$x + 5y = 3$$
$$5x + 12y = 5.$$
   (a) (1 point) Rewrite the above system as an equation $A\vec{v} = \vec{b}$ for a matrix A and vector $\vec{b}$.

   (b) (2 points) Explain why A is invertible and compute A^{-1}.

   (c) (2 points) Use (b) to solve the system of equations.

3. (10 point) For each of the following 2×2 matrices determine the following three pieces of information:

   (i) the characteristic polynomial of the matrix

   (ii) all eigenvalues of the matrix

   (iii) the eigenspace of each eigenvalue from (ii).

   (a) $$A = \begin{pmatrix} 3 & 1 \\ -1 & 1 \end{pmatrix}$$

   (b) $$B = \begin{pmatrix} 1 & 6 \\ 5 & 2 \end{pmatrix}$$

4. Let $P_{\vec{w}}$ be projection onto the line $y = 2x$. On Problem Set 8 you determined the standard matrix for this transformation.

   (a) (3 points) Show that 0 and 1 are eigenvalues for $P_{\vec{w}}$.

   (b) (3 points) Find all eigenvectors for both eigenvalues in (a).

   (c) (3 points) Show that 0 and 1 are all the eigenvalues of $P_{\vec{w}}$.

5. DON'T TURN IN. Let $A = \begin{pmatrix} 2 & -1 \\ 5 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ -3 & -6 \end{pmatrix}$.

(a) What is A^{-1}?

(b) Use (a) to find a matrix M so that $AM = B$.

(c) Use (a) to find a matrix N so that $NA = B$.