
Math 215: Linear Algebra

PROBLEM SET 14 : DUE OCTOBER 6

(26 points) Make sure you are familiar with the Academic Honesty policies for this class, as detailed on the syllabus. All work is due on the given day by 3 PM Grinnell Time, or 7 PM if you LaTeX the assignment. **For this assignment, show all of your computational work.**

Make sure you are familiar with the Academic Honesty policies for this class, as detailed on the syllabus. All work is due on the given day by the time lecture starts.

1. For this problem let $V = \mathcal{M}_{2 \times 2}$ be the vector space defined in Example 4.1.4 in the textbook. For each of the following subsets W of V , either prove W is a subspace (prove it satisfies the three conditions of the definition) or explicitly explain why it is not.

(a) (3 points) $W = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{R} \text{ and } a = 1 \right\}$

(b) (3 points) $W = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{R} \text{ so that } b = 0 \text{ or } c = 0 \right\}$. (Remember “or” in math means one or both options can happen.)

2. (4 points) Let $V = P_3$, defined in the discussion immediately following definition 4.1.13 in your textbook, and let $W = \{f(x) \in V : f'(x) \in P_1\}$ where by $f'(x)$ we mean the first derivative of the function $f(x)$. Either prove W is a subspace of V or explicitly explain why it is not.

3. (a) (2 points) Suppose a system of linear equations has augmented matrix

$$\begin{pmatrix} 1 & 7 & 3 & -4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{pmatrix}.$$

Determine the solution set represented by this matrix.

- (b) (2 points) Suppose a system of linear equations has augmented matrix

$$\begin{pmatrix} -3 & 1 & 4 & 5 & 2 & 1 \\ 0 & 1 & -1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 3 & -2 \\ 0 & 0 & 0 & 0 & 1 & -2 \end{pmatrix}.$$

Determine the solution set represented by this matrix.

4. (6 points) Completely describe the set of solutions to the following system:

$$\begin{aligned}3x_2 - 6x_3 + 6x_4 + 4x_5 &= -5 \\3x_1 - 7x_2 + 8x_3 - 5x_4 + 8x_5 &= 9 \\3x_1 - 9x_2 + 12x_3 - 9x_4 + 6x_5 &= 15.\end{aligned}$$

5. (6 points) Completely describe the set of solutions to the following system:

$$\begin{aligned}x - y + 2z &= 4 \\2x + 3y - z &= 1 \\7x + 3y + 4z &= 7.\end{aligned}$$