Math 215: Linear Algebra

Problem Set 17 : Due October 16

(25 points) Make sure you are familiar with the Academic Honesty policies for this class, as detailed on the syllabus. All work is due on the given day by 3 PM Grinnell Time, or 7 PM if you LaTeX the assignment. Make sure you show all your calculations. **Continue to show all your row reductions.**

1. Determine the column space, Col(A) for the following matrices and then determine the rank of each. Show your work and explain your answer.

   (a) (3 points) $\begin{pmatrix} 1 & 2 & 1 \\ 2 & 6 & 3 \\ 0 & 2 & 5 \end{pmatrix}$

   (b) (3 points) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 2 & 4 & 8 \end{pmatrix}$

2. Let $T : \mathbb{R}^5 \to \mathbb{R}^4$ be a linear transformation with

$$[T]_{\varepsilon_5}^{\varepsilon_4} = \begin{pmatrix} 1 & 4 & 8 & -3 & -7 \\ -1 & 2 & 7 & 3 & 4 \\ -2 & 2 & 9 & 5 & 5 \\ 3 & 6 & 9 & -5 & -2 \end{pmatrix}.$$

   (a) (3 points) Determine the Null(T).

   (b) (3 points) Determine range(T).

   (c) (2 points) What is the rank and nullity of T?

3. (4 points) Explain why there is no injective linear transformation $T : P_4 \to \mathcal{M}_{2\times 2}$.

4. Let $A = \begin{pmatrix} 1 & 4 & 3 \\ -1 & -2 & 0 \\ 2 & 2 & 3 \end{pmatrix}$.

   (a) (3 points) Compute det(A).

   (b) (4 points) Is A invertible? If so, find its inverse. If not, say why not.

5. (DO NOT TURN IN.) Let A be an $m \times n$ matrix and let B and C be $n \times p$ matrices. Prove that $A(B+C) = AB + AC$. (Suggestion: The formula for general matrix multiplication is your friend!)

6. (DO NOT TURN IN.) Let $A = \begin{pmatrix} -1 & -3 & 0 & 1 \\ 3 & 5 & 8 & -3 \\ -2 & -6 & 3 & 2 \\ 0 & -1 & 2 & 1 \end{pmatrix}$.

   (a) Compute $\det(A)$.

   (b) Is A invertible? If so, find its inverse. If not, say why not.

7. (DO NOT TURN IN.) Let a, b, c, x, y, be arbitrary elements of $\mathbb{R}$. Define $A = \begin{pmatrix} a & b & c \\ a+x & b+x & c+x \\ a+y & b+y & c+y \end{pmatrix}$.

   Prove that $\det(A) = 0$ regardless of the values of a, b, c, x, and y.