Math 215: Linear Algebra

Problem Set 3 : Due September 7

(19 points) Make sure you are familiar with the Academic Honesty policies for this class, as detailed on the syllabus. All work is due on the given day by 3 PM Grinnell Time, or 7 PM if you LaTeX the assignment.

1. (4 points) Below is the proof of the statement

   For all $a \in \mathbb{Z}$ we have $2a^5 + 6a^3 - 4a + 3$ is odd.

   I have deleted some pieces of the proof. Either fill in each of the blank spaces to make the proof correct or write your own proof from scratch (following the conventions we've learned in class and which are talked about in the book). **Please underline or color differently the filled in blanks on your submitted answer if you choose to just fill in blanks.**

   ______________________________. Then $2a^5+6a^3-4a+3 =$ ____________________ Since ____________________ is an integer, then ____________________ by the definition of odd numbers. Since a was arbitrary, we have proven the result.

2. (6 points) Write the *contrapositive* of each of the following statements. Your final answer should not have any *not* in it. You should not discuss whether the statements are true or false as stated.

   (a) If $|x| \neq -x$ then $x \leq 0$.

   (b) For all real numbers a and b, if $a \neq 0$ and $b \neq 0$ then $ab \neq 0$.

   (c) Let a be an integer. If there exists an $m \in \mathbb{Z}$ so that $a = 4m + 1$ then a is odd. (Hint: To get rid of the $\neq$ symbol, maybe consider the number line.)

3. (5 points) Prove the following statement by (a) writing down its contrapositive and then (b) proving the contrapositive.

   Let $a \in \mathbb{Z}$. If $3a + 2$ is odd, then a is odd.

4. (4 points) Suppose A and B are subsets of some set U. Use the proof technique for showing one set is a subset of another and the definitions of union and intersection to prove the following.

   (a) $A \cap B \subseteq A$

   (b) $A \subseteq A \cup B$