Math 215: Linear Algebra

Problem Set 5 : Due September 11

(18 points) Make sure you are familiar with the Academic Honesty policies for this class, as detailed on the syllabus. All work is due on the given day by 3 PM Grinnell Time, or 7 PM if you LaTeX the assignment

1. (4 points) In class, we defined a function $f : A \to B$ to be surjective when *for all* $b \in B$ *there exists an* $a \in A$ *so that* $f(a) = b$.

   (a) Carefully write the negation of this statement.

   (b) Use (a) to prove that the function $g : \mathbb{Z} \to \mathbb{Z}$ satisfying $g(m) = 2m + 1$ is not surjective.

2. (4 points) In class, we defined a function $f : A \to B$ to be injective when *for all* $a_1, a_2 \in A$, *if* $f(a_1) = f(a_2)$ *then* $a_1 = a_2$.

   (a) Write the contrapositive of this definition. (We will sometimes use this as an alternative way to describe an injective function).

   (b) Negate the definition of injective from class (i.e. the one written above!).

   (c) Use (b) to prove that $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = x^2$ is not injective.

3. (4 points) For each of the following questions, be sure to explain your work.

   (a) Find an example of some $\vec{u} \in \mathbb{R}^2$ so that $\text{Span}(\vec{u})$ is the solution set of the equation $4x - 7y = 0$.

   (b) Find an example of a, b, and $c \in \mathbb{R}$ so that $ax + by = c$ has solution set $\text{Span}(\binom{3}{2})$.

4. (6 points) We will prove the following statement.

   Let $\vec{u} \in \mathbb{R}^2$. *If* $\vec{w} \in$ *Span*$(\vec{u})$ *then Span*$(\vec{w}) \subseteq$ *Span*$(\vec{u})$.

   I have set up the outline of the proof. You should either fill in the blanks or you may write your own proof from scratch, but it should look very similar to my outline. **If you fill in the blanks, please underline or color differently the filled in blanks on your submitted answer.**

   We will prove this by assuming that $\vec{w} \in \text{Span}(\vec{u})$ and showing ____________________. To show containment of sets, we must show that for all ____________________ in $\text{Span}(\vec{w})$, $\vec{v}$ is also in ____________________.Let $\vec{v} \in \text{Span}(\vec{w})$ be arbitrary. Since $\vec{w} \in \text{Span}(\vec{u})$, we can ____________________. Since $\vec{v} \in \text{Span}(\vec{w})$ we can ____________________. Now notice that $\vec{v} =$ ____________________. Since ____________________ $\in \mathbb{R}$ we conclude that $\vec{v} \in \text{Span}(\vec{u})$. Since $\vec{v}$ was arbitrary, we have proven the containment, and so the original statement is true.