Math 215: Linear Algebra

Problem Set 6 : Due September 14

(18 points) Make sure you are familiar with the Academic Honesty policies for this class, as detailed on the syllabus. All work is due on the given day by 3 PM Grinnell Time, or 7 PM if you LaTeX the assignment

1. (4 points) In the previous assignment, you showed that $g : \mathbb{Z} \to \mathbb{Z}$ satisfying $g(m) = 2m + 1$ is not surjective. Now prove that $h : \mathbb{R} \to \mathbb{R}$ satisfying $h(x) = 2x + 1$ is surjective.

2. (4 points) Prove the following two sets are equal by showing that $A \subseteq B$ and that $B \subseteq A$. You should explicitly take an element in A and show it is in B and then take an element of B and show it is in A.

$$A = \left\{ \begin{pmatrix} 3 \\ 5 \end{pmatrix} + c \begin{pmatrix} 4 \\ 2 \end{pmatrix} : c \in \mathbb{R} \right\} \text{ and } B = \left\{ \begin{pmatrix} -1 \\ 3 \end{pmatrix} + c \begin{pmatrix} 4 \\ 2 \end{pmatrix} : c \in \mathbb{R} \right\}$$

   If you aren't sure where to start, take a specific element in A and play around with it to get it to look like an element in B. Then can you generalize your idea?

3. (4 points) (a) Is $\binom{2}{1}$ a linear combination of the vectors $\binom{1}{1}$ and $\binom{1}{2}$? Explain why or why not using the definition of a linear combination.

   (b) Is $\binom{0}{1}$ a linear combination of the vectors $\binom{1}{1}$ and $\binom{1}{2}$? Again, explain why or why not using the definition.

4. (6 points) For the following problem, explain which choice(s) of $x \in \mathbb{R}$ will make the following pairs of vectors $\vec{v_1}$ and $\vec{v_2}$ such that $\text{Span}(\vec{v_1}, \vec{v_2}) = \mathbb{R}^2$. Your answer should include *all* possible x values. Be sure to explain your answer in **complete sentence(s)**.

   (a) $\vec{v_1} = \binom{x}{1}, \vec{v_2} = \binom{4}{2}$

   (b) $\vec{v_1} = \binom{x}{x^2}, \vec{v_2} = \binom{-3}{9}$

   (c) $\vec{v_1} = \binom{x}{x^2}, \vec{v_2} = \binom{1}{x}$