Math 215: Linear Algebra

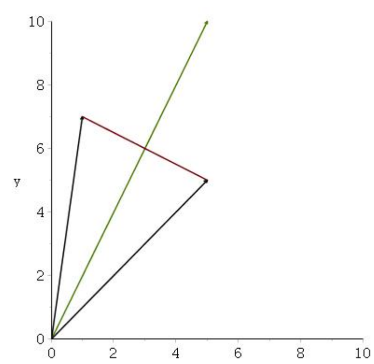
Writing Assignment 3 : Due September 24

Make sure you are familiar with the Academic Honesty policies for this class, as detailed on the syllabus. All work is due on PWeb on the given day by 3 PM or by 7 PM if you LaTeX it.

This is a writing assignment. You should treat this like you would a writing assignment in an English or Philosophy or History course, in the sense that everything you write should be part of a complete sentence and part of a larger paragraph which serves a clear purpose, your grammar and spelling should be accurate, and (if you handwrite it) you should not have crossed out sections where you change your mind about what you want say. For each problem you should plan out how you want to write it in an outline or draft, and then write a polished, final product to be submitted. Your audience should be fellow students who are excited about math but have not learned linear algebra yet. You are welcome to ask a friend who is not in the class to read your answer and let you know if it makes sense to them. There is no page or paragraph limits for this assignment but you should be (1) thorough and complete and (2) concise and exact.

1. In this question you will determine the standard matrix for the linear transformation $S : \mathbb{R}^2 \to \mathbb{R}^2$ given by $S(\vec{v})$ *is the vector* $\vec{v}$ *reflected across the line* $y = 2x$. Unless noted otherwise, be sure to explain your work in complete sentences.

   The problem will refer to the following picture. The green line is $y = 2x$ and the two black vectors are reflections across the line $y = 2x$. Think of one as $\vec{v} = \binom{x}{y}$ and the other as $S(\vec{v}) = \binom{a}{b}$. The key observation is that the midpoint of the line



   segment from $\vec{v}$ to $S(\vec{v})$ is the projection of $\vec{v}$ onto the line $y = 2x$. (Think about why this is true.) The red line in the picture represents the above mentioned line segment.

   (a) Determine the midpoint of this line segment in terms of $\vec{v}$ and $S(\vec{v})$. (Suggestion: Start by thinking of midpoints in $\mathbb{R}$. What is the midpoint of the line segment from -1 to 5 or from 3 to 9? Can you describe these midpoints in relation to the endpoint values?)

(b) We said that the midpoint you computed in (a) should be equal to the projection of $\vec{v}$ onto the line $y = 2x$. Set your value from (a) equal to $P_{\vec{w}}(\vec{v})$ where Span($\vec{w}$) is the line $y = 2x$. Then solve the equation you have created for $S(\vec{v})$ and simplify it.

(c) Use (b) to write the standard matrix for S. (This does not need to be in a complete sentence.)

(d) Test your transformation S from (b). Does it send the vector $\binom{5}{5}$ to the vector $\binom{1}{7}$?

(e) **Geometrically** explain which well known transformation $S \circ S$ is. You should not use algebra to explain this.

2. In this problem, we will figure out how to describe rotation as linear transformations in a different way from Proposition 2.5.9.

   (a) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that sends a vector $\vec{v}$ to the vector rotated $\frac{1}{4}$ of the way (or $90°$ counterclockwise (or anticlockwise for you British English speakers out there).

   The picture below gives an arbitrary vector $\binom{x}{y}$ in blue, and, together with the blue dotted line and the x-axis, this forms the triangle with corners $(0, 0)$, $(x, 0)$, and (x, y). When we rotate the blue triangle counterclockwise we get the red triangle. The red vector will be $T(\binom{x}{y})$. Use the triangles to determine what that output vector of this linear transformation would be, i.e. define the linear transformation $T(\binom{x}{y})$ as a linear transformation of the form in Proposition 2.4.3. Explain your process thoroughly.

   (b) Repeat the process from (a) with rotation $\frac{3}{4}$ of the way counterclockwise (so $270°$ degrees).

   (c) What problems do you run into if you try the same procedure for rotation counterclockwise by $\frac{1}{8}$ (or $45°$)? How about rotation $\pi/5$ radians counterclockwise?