Math 218: Elementary Number Theory

Homework 2 : Due September 5

§1.1 # 14. Let a_0, a_1, and a_2 be integers.

(a) Prove that if x is an integer and $a \mid x$ then $a \mid a_1x + a_2x^2$.

(b) Use (a) to prove that if x is a nonzero integer such that $a_2x^2 + a_1x + a_0 = 0$ then $x \mid a_0$.

(c) Based on (b), what integers would you have to try if you wanted to find an integer solution to the equation $x^2 + 4x + 3 = 0$? Why only those integers?

(d) Generalize (b)-(c) for the case of a polynomial equation of degree n with a_i the integer coefficients, i.e. suppose that $a_nx^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0 = 0$. Now what can you say about potential integer solutions x?

§1.2 # 7 and 13. Remember to always explain why your answers are correct!

(a) Consider the table on page 8 of the book. For what integers is $\tau(n)$ odd? What common property do those integers share?

(b) Write a definition for prime numbers which uses the function $\sigma(n)$. In particular, your definition should apply to all prime numbers and no composite numbers.

§1.3 #4. Prove that 3 divides $2^{2n} - 1$ for every positive integer n.

§1.3 # 7. If $x \neq 1$ prove that for every positive integer n

$$\frac{1 - x^n}{1 - x} = 1 + x + \cdots + x^{n-1}.$$

Additional #1. A chocolate bar consists of n squares arranged in a rectangular pattern. You split the bar into small squares, always breaking along the lines between the squares. Use induction to prove that no matter how you break the bar, it takes $n-1$ breaks to split it into the n smaller squares.

Comment: Chocolate bars are not necessarily one long line of rectangles. When $n = 6$ the bar could consist of 6 small squares in a row, or it could consist of two rows of 3 squares each.

Here is a picture of a chocolate bar, and some physics on why they typically break at the seams: `http://physics.stackexchange.com/questions/238202/why-do-chocolate-bars-usually-break-at-the-cleavages`