Math 218: Elementary Number Theory

Homework 9 : Due October 10

2.2 #11. (a) If p is a prime, prove that the binomial coefficient $\binom{p}{r} \equiv 0 \bmod p$ for $r = 1, 2, 3, \ldots, p-1$.
(b) Use (a) to prove that $(a+b)^p \equiv a^p + b^p \bmod p$.

1. (a) Prove that
$$3^n = \sum_{k=0}^{n} \binom{n}{k} 2^k.$$
(b) For $n \geq 1$, prove
$$\binom{n}{0} - \binom{n}{1} + \cdots + (-1)^k \binom{n}{k} + \cdots + (-1)^n \binom{n}{n} = 0.$$

2.2 #13. As a kid, you likely learned that 9 divides a number n if and only if the sum of the digits of n is divisible by 9. In this problem, you are proving why that divisibility test works. See the problem writeup in the book and note that the a_i in the book represent digits between 0 and 9. The first sentence of the problem is telling you another statement you may remember from grade school: that the number "23491" means $2 \cdot 10^4 + 3 \cdot 10^3 + 4 \cdot 10^2 + 9 \cdot 10^1 + 1 \cdot 10^0$.

2.3 #6. If a is a unit in Z_m, prove that $m - a$ is also a unit in Z_m.

2.3 #14. Let p and q be odd primes. Which $a \in Z_{pq}$ are such that $a^2 \equiv 1 \bmod pq$? (*Suggestion*: there are several different cases to consider.)