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# Math 218: Elementary Number Theory

## INDUCTION PROBLEMS

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1. Prove by induction that for every positive integer  $n$ , the value  $n^3 + 2n$  is divisible by 3.
2. Prove by induction on  $n \geq 5$  that  $2^n > n^2$ .
3. Prove by induction that for every positive integer  $n$  with  $a \neq 1$

$$\sum_{i=1}^n ia^i = \frac{na^{n+2} - (n+1)a^{n+1} + a}{(a-1)^2}.$$

4. Suppose  $M_i$  is an  $r_{i-1} \times r_i$  matrix for  $1 \leq i \leq n$ . Prove by induction that for all positive integers  $n$ , the matrix product  $M_1 \cdot M_2 \cdots M_n$  is an  $r_0 \times r_n$  matrix.